

Gaussian bosonic synergy: quantum communication via realistic channels of zero quantum capacity

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As with classical information, error-correcting codes enable reliable transmission of quantum information through noisy or lossy channels. In contrast to the classical theory, imperfect quantum channels exhibit a strong kind of synergy: there exist pairs of discrete memoryless quantum channels, each of zero quantum capacity, which acquire positive quantum capacity when used together. Here we show that this “superactivation” phenomenon also occurs in the more realistic setting of optical channels with attenuation and Gaussian noise. This paves the way for its experimental realization and application in real-world communications systems.

In the spirit of Shannon’s information theory [1], any physical process acting on a quantum system can be thought of as a communication channel. One may then speak of its capacity for transmitting quantum states as the fundamental amount of quantum information that can be protected using error correction. Quantum capacity measures the number of qubits, or two-level quantum systems, that can be protected with vanishing error in the limit of many channel uses. In contrast to Shannon’s capacity, no simple formula is known for the quantum capacity. While finite-dimensional systems like qubits are convenient units for quantifying quantum information and for describing abstract protocols, real-world applications require consideration of continuous-variable systems, such as optical, electromagnetic, and more general bosonic systems.

Photons are the natural carriers of information in radio, cellular, fiber and free-space optical networks. Since bosons mediate the fundamental forces of nature, understanding information flows in bosonic systems is of both deep theoretical and practical importance. Noise in conventional networks is often well-approximated by additive gaussian noise, and the six decades since Shannon’s theory was introduced have seen the emergence of a mature theory of communication in such practical networks. Low power optical noise, on the other hand, is quantum mechanical in nature. Despite considerable recent progress, comparatively simple questions about point-to-point capacities of optical channels with gaussian noise, or even finite-dimensional channels, remain unanswered. Nonetheless, commercial quantum networks are being deployed worldwide for fundamentally quantum tasks like quantum cryptography [2], while experimental development of quantum memories paves the way toward the quantum repeaters and quantum computers of the future.

Gaussian quantum noise is a generalization of the additive white gaussian noise at the heart of classical information theory [3]. It arises when optical modes unitarily interact via a quadratic Hamiltonian with vacuum environment modes [4, 5]. Alternatively, a gaussian state is a state with a gaussian characteristic function and a gaussian channel maps gaussian states to gaussian states. Examples of gaussian states include thermal, coherent, and squeezed states. Fock, or number states, are not gaussian and are much more difficult to produce. Table 1 describes gaussian states and channels in detail.

Any noisy quantum evolution $\mathcal{N}: A \rightarrow B$ from states on \mathcal{H}_A to states on \mathcal{H}_B can be mathematically extended to a unitary interaction of an input state with an uncorrelated and inaccessible environment with Hilbert space \mathcal{H}_E . This defines a second, complementary channel to the environment \mathcal{H}_E , that appears in a useful lower bound to the quantum capacity. This lower bound is the maximum of the *coherent information*, or difference $H(B) - H(E)$ between the entropies of the output and environment, maximized over all input states

	General State	Gaussian State
Characteristic Function	$\text{Tr } \rho e^{\mathbf{v}^t J \mathbf{R}}$ where $\mathbf{R} = (Q_1 \ P_1 \ \cdots \ Q_m \ P_m)^t$	$e^{\mathbf{v}^t \mathbf{d} - \frac{1}{4} \mathbf{v}^t \gamma \mathbf{v}}$ with $d_i = \langle R_i \rangle$ and $\gamma_{ij} = \langle R_i R_j + R_j R_i \rangle - d_i d_j$
Uncertainty Relations	$[R_j, R_k] = i J_{jk}$ where $J = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}^{\oplus n}$	$\gamma + iJ \geq 0$
Noisy Evolution	$\rho \rightarrow \text{Tr}_E U \rho \otimes 0\rangle\langle 0 U^\dagger$ U unitary	$\gamma \rightarrow X \gamma X^t + Y$ $Y = Y^t$ and $Y + i(J - X J X^t) \geq 0$

Table 1: General vs Gaussian bosonic states. The quantum phase space A of m bosonic modes is described by canonical coordinates $q_1, p_1, \dots, q_m, p_m$. A corresponding vector of canonical operators \mathbf{R} acts on the underlying infinite-dimensional quantum Hilbert space \mathcal{H}_A and satisfies Heisenberg's uncertainty relations $[R_j, R_k] = i J_{jk}$. A state of A is gaussian precisely when its characteristic function is a gaussian function of the phase space vector \mathbf{v} . Such a state is characterized by its displacement vector \mathbf{d} and covariance matrix γ , both defined in terms of expectation values $\langle R_i \rangle = \text{Tr}(R_i \rho)$ and $\langle R_i R_j \rangle = \text{Tr}(R_i R_j \rho)$. Some examples of covariance matrices are the covariance of the vacuum, \mathbb{I}_2 , a single-mode thermal (Bose-Einstein) state with average photon number n which has covariance matrix $(2n+1)\mathbb{I}_2$, as well as a squeezed vacuum state with $\gamma = \text{diag}(\eta, 1/\eta)$. Disregarding phase space displacements, gaussian channel can be described by a linear map from covariance matrices on a set of input modes A , to a set of output modes B .

on \mathcal{H}_A , where the von Neumann entropy of a density matrix ρ is $H = -\text{Tr } \rho \log_2 \rho$. It is a lower bound in the sense that good sequences of error-correcting codes achieving this rate exist [6, 7, 8]. Only in special cases can we effectively calculate the optimization implicit in the coherent information, let alone the quantum capacity itself.

Quantum capacities of gaussian channels have been considered in detail [4, 9], where coherent information was calculated to give a lower bound for several examples. The quantum capacity of the lossy bosonic channel was found in [10]. Classical and cryptographic capacities of bosonic gaussian channels have also been considered by many authors [4, 9, 11, 10, 12]. However, even restricting to gaussian noise does not appear to make the problem solvable and the potential exists for exotic behavior for these channels.

Pairs of zero quantum capacity channels can nevertheless allow noiseless communication when used together [13]. This *superactivation* arises when two zero-capacity channels have very different noise properties. The weakness of each channel is overcome by the strengths of the other, illustrating that the communication capability of a channel is not a simple function of the channel alone, but also depends on the context in which it is used. The discovery of [13] followed a sequence of superadditivity findings in the multi-party and two-way settings [14, 15, 16, 17], and was followed by several substantial discoveries in the conventional one-way setting [18, 19, 20, 21, 22]. Below we present some simple and natural examples of superactivation with gaussian channels that can potentially be realized with current technologies, demonstrating the richness of the set of gaussian channels and the complexity of their capacity-achieving protocols. Superactivation is therefore not merely an oddity confined to unrealistic models but is in fact *necessary* for a proper characterization of realistic communication settings.

There are two classes of channels known to have zero quantum capacity. The first is the antidegradable channels [23], where the environment can simulate the output. A simple example is a 50% attenuation channel,

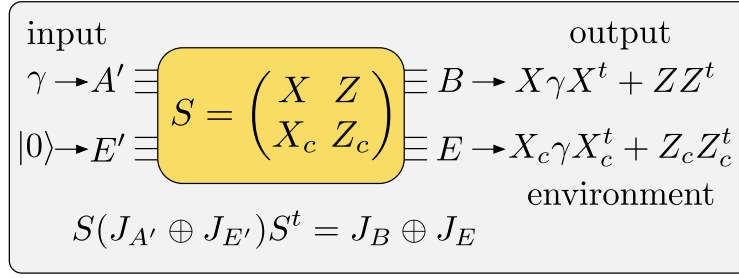


Figure 1: Just as an arbitrary quantum channel can be implemented as a unitary transformation acting on a larger space, every gaussian quantum channel can be represented by a symplectic matrix S as illustrated as a block matrix in the figure. The corresponding channel maps the input A' to the output B while the complement maps it to the environment E . The ancillary input modes E' are assumed to be in the vacuum state with covariance $\mathbb{I}_{E'}$. The matrix S is required to be symplectic, or canonical transformation, meaning that it satisfies the equation at the bottom of the figure, and is thus compatible with the symplectic structures of the input and output modes.

which is modeled by a beamsplitter. The other zero-capacity class is the entanglement binding channels [24], which produce a weak type of entanglement between sender and receiver that prohibits quantum communication and is analogous to thermodynamical bound energy. These PPT channels only produce states satisfying the positive partial transpose (PPT) nondistillability criterion [25, 26] which, for covariance matrix γ_{AB} is $\gamma_{AB} + i(J_A \oplus -J_B) \geq 0$ [27]. These are precisely the channels that remain physical when composed with the potentially nonphysical operation of time-reversal. A gaussian channel of the form $\gamma \rightarrow X\gamma X^t + Y$ is PPT if and only if $Y + i(J + XJX^t) \geq 0$.

Since PPT bound entangled gaussian states can only occur when each side has at least two modes [27], the smallest PPT channel one might hope to superactivate would act on two modes. We have found a family of such examples, the simplest being

$$X = \begin{pmatrix} -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}, \quad Y = \begin{pmatrix} \sqrt{2} & 0 & 0 & 0 \\ 0 & \sqrt{2} & 0 & 0 \\ 0 & 0 & \sqrt{2} & 0 \\ 0 & 0 & 0 & \sqrt{2} \end{pmatrix}. \quad (1)$$

Direct calculation reveals that each of the matrices $Y + i(J \pm XJX^t)$ has eigenvalues $\{0, 0, 2\sqrt{2}, 2\sqrt{2}\}$, so this indeed represents a physical PPT map. Combining the channel (1) with a 50% attenuation channel results in a channel acting on three-mode covariance matrices as

$$\gamma \mapsto \left(X \oplus \frac{1}{\sqrt{2}}\mathbb{I}_2\right) \gamma \left(X \oplus \frac{1}{\sqrt{2}}\mathbb{I}_2\right)^t + Y \oplus \frac{1}{2}\mathbb{I}_2. \quad (2)$$

In the appendix, we show how to derive the action of the complementary channel using a symplectic representation as in Figure 1. Then we describe a family of three-mode covariance matrices that achieve .05 bits of coherent information at input power ≈ 60 photons/channel use, and over .06 bits with input power ≈ 812 photons/channel use.

Because the matrix $Y + i(J + XJX^t)$ is not full rank, the channel (1) is on the boundary of the PPT channels and any claim of superactivation will be sensitive to experimental errors. In Figure 2, we present a family of symplectic transformations that include (1) as a special case and are otherwise experimentally robust. As any symplectic transformation can be implemented physically as a combination of passive linear optical elements (beam splitters and phase shifters) together with single-mode squeezing, and we present our examples as circuits of this sort. In Figure 3, we show the positive coherent information generated by the examples of Figure 2 for a range of squeezing parameters and for reasonable input powers, supporting the notion that that superactivation is indeed generic. While linear optics are straightforward to implement in a laboratory, the nonlinear squeezings required by our examples will present more of a challenge. It is nonetheless possible to generate squeezing on

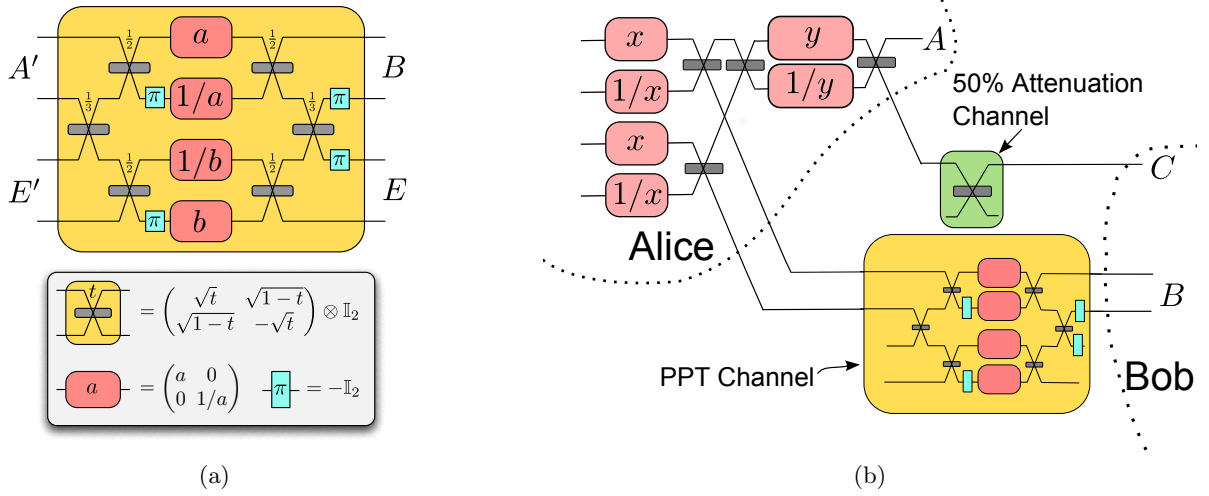


Figure 2: A two-parameter family of optical circuits implementing a symplectic transformation in the sense of Figure 1. For a range of parameters, we will see that the associated channel is in the interior of the set of PPT channels. At the bottom, we give the explicit symplectic transformation associated to the three building blocks of our circuits – transmissivity $1 - t$ beamsplitters, nonlinear single-mode squeezing, and half-wave phase plates. The example (1) is related to the channel with $(a, b) = \left(\sqrt{3} + \sqrt{2}, \frac{\sqrt{3}+1}{\sqrt{2}}\right)$ by a canonical transformation. (b) A family of circuits using the PPT and attenuation channels to generate coherent information between the purification A of the input and the channel outputs BC . Each beamsplitter has parameter $t = \frac{1}{2}$. We see below that superactivation is possible for a flexible range of parameters.

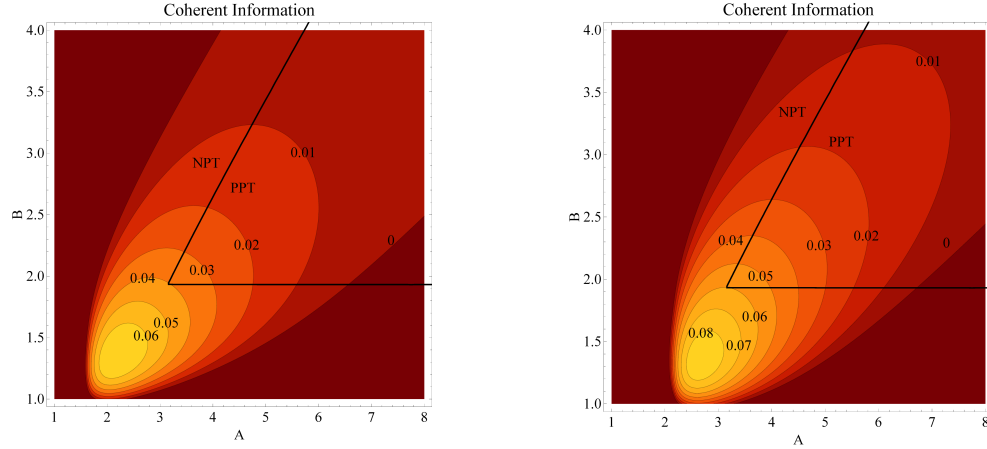


Figure 3: Superactivation for a wide range of parameters. Each plot shows the coherent information generated by the family of channels of Figure 2(a) using the strategies of Figure 2(b). Superactivation occurs in the triangular region to the upper right where the channels are PPT. The corner of this region is at $(a, b) = \left(\sqrt{3} + \sqrt{2}, \frac{\sqrt{3}+1}{\sqrt{2}}\right)$ and corresponds via symplectic transformations to the example (1). The plot in (a) shows the coherent information for moderate values of squeezing, $x = 3$ and $y = 3$. The plot in (b) is similar but for large $x = 20$ and optimal value $y = 2 + \sqrt{3}$.

the order of 10dB with current technology [28], corresponding to the map $(P, Q) \rightarrow (\sqrt{10}P, \frac{1}{\sqrt{10}}Q)$, and our examples generally require squeezing of this order. Although an example using only linear optical elements would be desirable, we suspect, but cannot prove, that none exist.

We can also analyze how our channels can arise from the continuous interaction between transmission and environment modes. This raises the possibility of our channel occurring “in the wild.” Since quadratic Hamiltonians are ubiquitous, it could be that real-world optical systems require using superactivation to achieve optimal performance. For the purpose of illustration, Figure 4 shows a natural Hamiltonian that implements

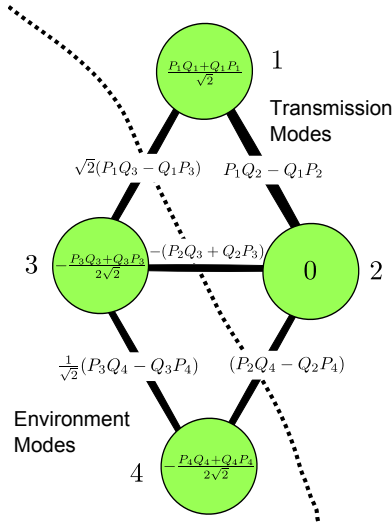


Figure 4: Generating Hamiltonian. The full Hamiltonian is the sum of the individual and pairwise interaction terms shown. If environment modes 3 and 4 begin in the vacuum state and this interaction runs for a time π , the resulting noisy evolution of transmission modes 1 and 2 is given by the $(a, b) = \left(\sqrt{3} + \sqrt{2}, \frac{\sqrt{3}+1}{\sqrt{2}}\right)$ channel from Figure 2.

the channel in this family with $(a, b) = \left(\sqrt{3} + \sqrt{2}, \frac{\sqrt{3}+1}{\sqrt{2}}\right)$ after evolution for a time π .

Even before the superactivation result of [13], it was known that the existence of bound-entangled states with negative partial transpose (NPT) would imply superactivation of distillable entanglement [16]. To date no such NPT quantum states or channels have been found. But it is known that there are no NPT bound entangled gaussian channels [27]. This might have suggested that the gaussian channels would be too simple for superactivation occur. As we have shown, this is not the case. In [13], superactivation was shown to be a consequence of the existence of PPT channels with private capacity [29, 30]. Rather than pursuing this idea, our approach has been to demonstrate superactivation directly. Notably, we don't know whether our channels have any private capacity or, for that matter, whether there are any PPT gaussian channels with positive private capacity.

Our results show that, far from a purely mathematical or singular phenomenon, superactivation arises naturally for a range of parameters in gaussian bosonic systems. Because it occurs in systems that seemed to be too noisy to be useful, superactivation points to the possibility of powerfully enhanced error correction for quantum memories and repeaters in the very noisy regime. It also unveils an unforeseen complexity in the theory of quantum mechanics with gaussian states.

Acknowledgments

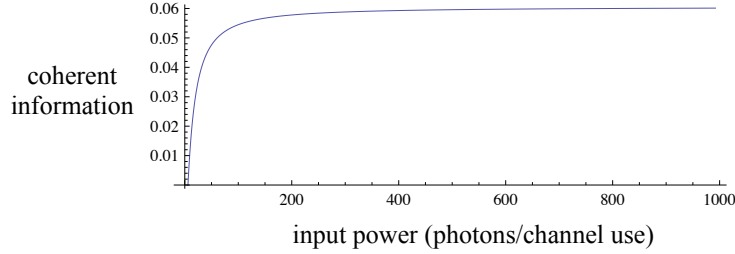
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Appendix

Evaluating the coherent information over the channel (1) on the family of gaussian states with covariance

$$\gamma = \begin{pmatrix} 7 \cosh(c) & 0 & \alpha_+ \sinh(c) & 0 & \alpha_- \sinh(c) & 0 \\ 0 & 7 \cosh(c) & 0 & -\alpha_+ \sinh(c) & 0 & \alpha_- \sinh(c) \\ \alpha_+ \sinh(c) & 0 & \sqrt{2} \cosh(c) & 0 & \cosh(c) & 0 \\ 0 & -\alpha_+ \sinh(c) & 0 & \sqrt{2} \cosh(c) & 0 & -\cosh(c) \\ \alpha_- \sinh(c) & 0 & \cosh(c) & 0 & \sqrt{2} \cosh(c) & 0 \\ 0 & \alpha_- \sinh(c) & 0 & -\cosh(c) & 0 & \sqrt{2} \cosh(c) \end{pmatrix}, \quad (3)$$

where $\alpha_{\pm} = \sqrt{\frac{7}{\sqrt{2}} + 2\sqrt{3} \pm \frac{1}{2}}$, gives superactivation for all positive input powers, as illustrated here:



In particular, this achieves .05 bits at $c = 3.19$, or at input power of about 60 photons/channel use, and achieves .06 bits at $c = 5.8$, or about 800 photons/channel use. To compute the coherent information, one also requires an expression for the complementary channel to (1). This can be obtained from a symplectic extension in the sense of Figure 1. A particularly simple extension exists in this case, given by the block-diagonal symplectic matrix

$$\begin{pmatrix} X & Z \\ Z & X \end{pmatrix}, \quad \text{where} \quad Z = \begin{pmatrix} \beta_+ & 0 & \beta_- & 0 \\ 0 & \beta_- & 0 & \beta_+ \\ \beta_- & 0 & -\beta_+ & 0 \\ 0 & \beta_+ & 0 & -\beta_- \end{pmatrix},$$

and $\beta_{\pm} = \sqrt{\frac{1}{\sqrt{2}} \pm \frac{1}{2}}$, with X as given in (1).

Computation of the coherent information requires computing the von Neumann entropy $H(\rho) = -\text{Tr} \rho \log_2 \rho$ of a gaussian state ρ . For a state of m modes, this can be computed starting with the covariance matrix γ as follows. The eigenvalues of $J\gamma$ come in complex conjugate pairs $\pm i\lambda_j$, where the λ_j are called the symplectic eigenvalues of γ . There then exists a symplectic matrix S such that

$$S\gamma S^t = \lambda_1 \mathbb{I}_2 \oplus \cdots \oplus \lambda_m \mathbb{I}_2.$$

The von Neumann entropy for such states has the simple form

$$H(\rho) = \sum_j \left(\frac{\lambda_j + 1}{2} \right) \log \left(\frac{\lambda_j + 1}{2} \right) - \left(\frac{\lambda_j - 1}{2} \right) \log \left(\frac{\lambda_j - 1}{2} \right).$$

The channel described in (1) is related to the $(a, b) = \left(\sqrt{3} + \sqrt{2}, \frac{\sqrt{3}+1}{\sqrt{2}} \right)$ point in our family of examples as follows. The latter channel is explicitly described by the matrices

$$X' = \begin{pmatrix} \sqrt{2} & 0 & 1 & 0 \\ 0 & -\sqrt{2} & 0 & 1 \\ -1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \end{pmatrix} \quad Y' = \begin{pmatrix} 2 & 0 & -\sqrt{2} & 0 \\ 0 & 2 & 0 & \sqrt{2} \\ -\sqrt{2} & 0 & 2 & 0 \\ 0 & \sqrt{2} & 0 & 2 \end{pmatrix}. \quad (4)$$

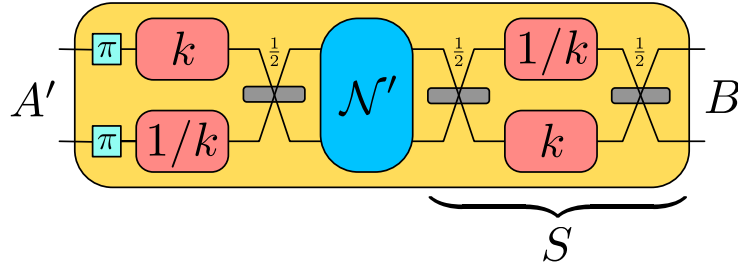


Figure 5: The channel (1) is related to the channel \mathcal{N}' at the point $(a, b) = \left(\sqrt{3} + \sqrt{2}, \frac{\sqrt{3}+1}{\sqrt{2}}\right)$ in Figure 2 by symplectic transformations at the input and output, pictured above with $k = \sqrt{\sqrt{2}-1}$. As these correspond to Hilbert space unitaries, the capacity properties of the channels are the same. The matrix S in (5) corresponds to the two-mode squeezing primitive at the output of the channel \mathcal{N}' .

These are related to the matrices (1) by the transformation $X = -SX'S^{-1}T$, $Y = SY'S^t$, where T is the matrix of a 50%-beamsplitter and

$$S = \begin{pmatrix} \beta_+ & 0 & \beta_- & 0 \\ 0 & \beta_+ & 0 & -\beta_- \\ \beta_- & 0 & \beta_+ & 0 \\ 0 & -\beta_- & 0 & \beta_+ \end{pmatrix}, \quad (5)$$

is a symplectic transformation corresponding to a two-mode squeezing operation. The matrix S can be viewed as diagonalizing the noise term Y' and is responsible for the simple form of the example (1).

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